Math 221: Test 1 Review Sheet

This document tells you everything that you will need to know in order to do well on your first test. Each bullet is a requirement that you should ensure that you satisfy prior to taking the test. Not all bullets have problems following them, but these concepts are just as important.

- Know the the definitions of the terms below. There will be one definition matching question on the test, so being able to recognize the definition is good enough. However, do not expect that the definitions will be worded the same.

  Definitions:

  1. Set - Any collection of objects with no repetitions.
  2. Element - An object within a set.
  3. Cardinal Number - The number of elements of a set.
  4. Empty Set - The set with no elements.
  5. Universal Set - The set of all elements being considered in a given discussion.
  6. Complement - The set of elements in the universal set that are not in the referenced set.
  7. Subset - A set in which each of its elements are contained in a second set.
  8. Proper Subset - A set which is a subset of a second set but is not equal to it.
  9. Intersection - The set of elements which are in both of the referenced sets.
  10. Union - The set of elements which are in either of the referenced sets.
  11. Set Difference - The set of elements which are in the first set but not in the second set.

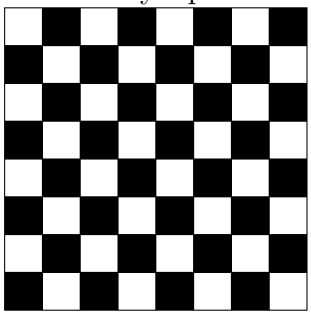
1.1

- State and briefly explain each of the 4 steps of the problem solving process.
- Solve word problems related to the problems done in class and in homework.

  Problem Solving: Solve the following problems. These are all of the problems you were given as examples, and the test questions will be similar.

  1. If you create a book out of n sheets, what is the sum of the page numbers?
  2. What is the sum of the first n natural numbers?
  3. What is the sum of the even numbers less than or equal to 40?
  4. If there are 10 people in the room and each person shakes every other person's hand, how many handshakes were performed? Nobody shakes hands twice, and shaking your own hand would make you look weird...
  5. A wealthy family hired a maid and a gardener. The maid comes in every 2 days, and the gardener comes in every 3 days. If they started on the same day, how many days will go by before they come in on the same day again?
  6. To get some cardio exercise, you climb the stairs in a tall building. You start from the first floor (Floor 1). You then go up 3 floors, down 2 floors, up 7 floors, down 5 floors, and then up 7 floors to stop at the top floor. How many floors does the building have?
  7. Find two numbers whose product is 42 and whose sum is 17.
  8. You have an 80 average on 6 quizzes. Your teacher tells you that you can drop your lowest quiz grade of 30. What is your new average?
  9. A farmer has a daughter who needs more practice in mathematics. One morning, the farmer looks out in the barnyard and sees a number of pigs and chickens. The farmer says to her daughter, "I count 24 heads and 80 feet. How many pigs and how many chickens are out there?"
  10. Arrange the numbers 1 through 9 into a square subdivided into nine smaller squares, so that the sum of every row, column, and main diagonal is the same.

11. Take any number and add 15 to the number. Now multiply that sum by 4. Next subtract 8 and divide the difference by 4. Now subtract 12 from the quotient and write down the answer. Your professor can tell you the original number. How is your professor find the original number so quickly?
12. How many squares can be found in the 8 by 8 checkerboard figure below?



13. Andrew, Michael, and Travis played a strategy game. Michael did not come in first place, as usual. Travis beat Michael, but he did not come in first place. Who took first, second, and third place?
14. If two people won 3 games of checkers each, what is the minimum number of games played?
15. Andrew, Danny, Heather, and Michael went to an undisclosed location to conduct the secret business of deciding what to eat for lunch the next day. They arrived at 9:55 PM, 10:00 PM, 10:10 PM, and 10:15 PM. Using the clues below, determine who arrived at each time.
    - Danny thought he was being followed, so he circled around the block several times and ended up arriving at 10:10 PM.
    - Heather arrived 15 minutes before Michael.

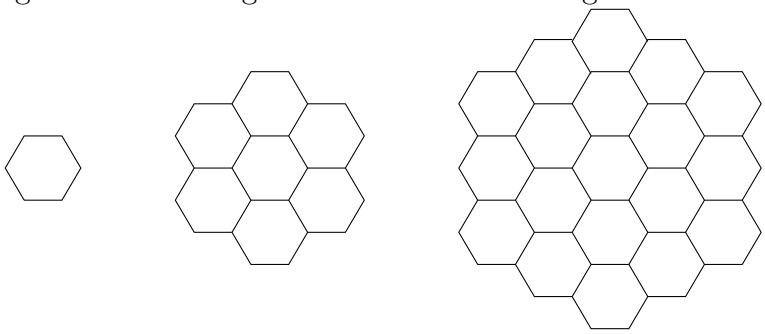
<u>1.2</u>

- Explain why patterns are not fool proof.
- Recognize and interpolate (find the next term) patterns of numbers and/or pictures.

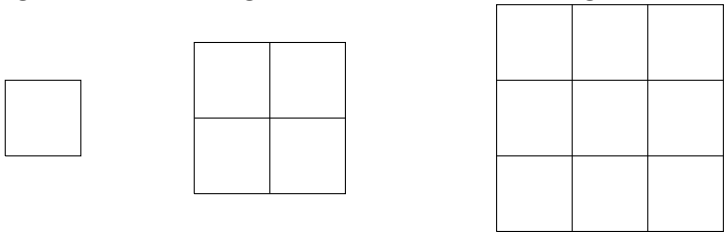
Patterns: Determine the next term in the following patterns.

1. 3, 8, 13, 18, ___
2. 0, 3, 8, 15, ___

3. Figure 1 Figure 2 Figure 3



4. Figure 1 Figure 2 Figure 3



2.1

- Given a setup of blocks, flats, longs, and units, write the number correctly in base five or base ten. You may use your base 5 or the base 10 blocks on the test.

  Base 10 Numbers: Given a setup of blocks, flats, longs, and units **in base 10**, perform the exchanges to write the number correctly in base 10.

  1. 1 block, 12 flats, 10 longs, 16 units
  2. 0 blocks, 7 flats, 8 longs, 26 units
  3. 2 blocks, 8 flats, 18 longs, 34 units

  Base 5 Numbers: Given a setup of blocks, flats, longs, and units **in base 5**, perform the exchanges to write the number correctly in base 5. Make sure that you include the subscript "five" after your number, or it is incorrect.

  1. 1 block, 4 flats, 8 longs, 7 units
  2. 0 blocks, 7 flats, 8 longs, 19 units
  3. 2 blocks, 6 flats, 14 longs, 30 units

- Convert between base five and base ten.

  Converting Base 5 to Base 10: Convert the following numbers from base 5 to base 10.

  1. 304_{five}
  2. 1342_{five}
  3. 2140_{five}
  4. 4321_{five}

  Converting Base 10 to Base 5: Convert the following numbers from base 10 to base 5.

  1. 400_{ten}
  2. 61_{ten}
  3. 486_{ten}
  4. 276_{ten}
  5. 502_{ten}
  6. 256_{ten}
  7. 219_{ten}
  8. 26_{ten}

- Count the numbers in base five.

  Counting Base 5 Numbers: Write the first 125 base five numbers.

2.2

- Use set builder notation to represent sets. You will be expected only to read them, but practicing writing them will make this easier.

  Set Builder Notation: Use set builder notation to write the following sets. There may be more than 1 correct answer.

  1. The set of all negative even integers.
  2. $\{1, 2, 6, 24, 120, ...\}$ (Hint: $6 = 3 \cdot 2 \cdot 1$, $24 = 4 \cdot 3 \cdot 2 \cdot 1$, etc.)
  3. The set of all odd multiples of 3, $\{3, 9, 15, ...\}$
  4. The set of real numbers greater than or equal to 3 and less than π.
  5. The set of numbers with integer square roots.
  6. The set of all sets containing one real number.

- Write the elements contained in $\mathbb{N}$ (natural numbers), $\mathbb{Z}$ (integers), $\mathbb{R}$ (real numbers).

- Give the cardinal numbers of sets.

  Cardinal Numbers: Determine the cardinal number of the following sets.

  1. $\{2, 4, 6, ..., 100\}$
  2. $\{-2, -1, 0, 1, 2\}$
  3. $\mathbb{R}$
  4. $\varnothing$
  5. $\{x \in \mathbb{R} \mid 0 < x < 1\}$
  6. $\{x^2 \mid x \in \mathbb{N} \text{ and } x \leq 5\}$

- Give the complements of sets, given a universal set.

  Complements: Given $U = \{1, 2, 3, ..., 10\}$, find the complements of the following sets.

  1. $A = \{2, 4, 6, 8, 10\}$
  2. $B = \{1, 2, 3, ..., 10\}$
  3. $C = \{1, 3, 7\}$

- Fill in the blanks like the examples in class with $\in$, $\notin$, $\subseteq$, and $\nsubseteq$, including problems with $\mathbb{N}$, $\mathbb{Z}$, and $\mathbb{R}$.

  Elements and Subsets: Fill in the blank with $\in$, $\notin$, $\subseteq$, $\nsubseteq$. If the blank is filled with $\subseteq$, then also tell whether you could also put $\subset$ or $=$.

  1. $\{0, 2, 4, 6\}$ ___ $\{0, 2, 4\}$
  2. 0 ___ $\{x^2 \mid x \in \mathbb{N}\}$
  3. $\varnothing$ ___ $\varnothing$
  4. $\varnothing$ ___ $\{0, 2, 4, 6\}$
  5. $\sqrt{-1}$ ___ $\mathbb{R}$
  6. $\{2^n \mid n \in \mathbb{N}\}$ ___ $\{2n \mid n \in \mathbb{N}\}$
  7. $\mathbb{N}$ ___ $\mathbb{N}$
  8. $\{0, 2, 4, 6\}$ ___ $\{2x \mid x \in \mathbb{Z}\}$
  9. $\{0, 2, 4, 6\}$ ___ $\varnothing$
  10. 8 ___ $\{0, 2, 4, 6, ...\}$
  11. $\{0, 2, 4\}$ ___ $\{0, 2, 4, 6\}$
  12. -1 ___ $\mathbb{N}$
  13. 0 ___ $\{0, 2, 4, 6\}$
  14. $\varnothing$ ___ $\{\varnothing\}$
  15. -2 ___ $\varnothing$
  16. 8 ___ $\{0, 2, 4, 6\}$
  17. $\{0, 2, 4, 6\}$ ___ $\mathbb{N}$
  18. 5 ___ $\mathbb{Z}$
  19. $\{0, 2, 7\}$ ___ $\{0, 2, 7\}$
  20. $\mathbb{Z}$ ___ $\mathbb{R}$

2.3

- Find the union, intersection, and set difference of two given sets.

  Unions, Intersections, and Set Differences: Given $U = \{-10, -9, -8, ..., 10\}$, $A = \{2, 4, 6, 8, 10\}$, $B = \{1, 3, 5\}$, $C = \{2, 3, 4, 5, 6, 7\}$, and $D = \{1, 2, 3, ..., 10\}$ perform the following set operations.

  1. $A \cap C$
  2. $A \cup D$
  3. $D - A$
  4. $D \cup \varnothing$
  5. $B \cap A$
  6. $\overline{A} \cap B$
  7. $C - \overline{B}$
  8. $\overline{D} \cap A$
  9. $A - \mathbb{Z}$
  10. $\varnothing \cup \varnothing$
  11. $C \cap \mathbb{R}$
  12. $\varnothing - A$

- Fill in Venn Diagrams for unions, intersections, complements, set differences, and combinations of these.

Venn Diagrams: Draw the Venn Diagram for the following sets.

1.	$A \cap B$	2.	$A \cup B$	3.	$A - B$
4.	$\overline{A}$	5.	$\overline{A} \cup B$	6.	$\overline{A} - B$
7.	$A - \overline{B}$	8.	$\overline{A} \cap \overline{B}$	9.	$\overline{A} \cup \overline{B}$
10.	$(A \cup B) \cup C$	11.	$A \cup (B \cap C)$	12.	$(A \cup B) \cap (A \cup C)$
13.	$A - (B \cap C)$	14.	$A \cap \left(\overline{B \cup C}\right)$	15.	$\overline{(A \cup B) \cup C}$

Helpful Rules/Formulas:

1. The Four-Step Problem-Solving Process is: (1) Understand the Problem, (2) Devise a Plan, (3) Carry out the Plan, (4) Look Back.
2. Various Strategies: Look for a pattern, make a table, draw a picture, examine a related problem, examine a simpler case, identify a sub-goal, make a diagram, guess and check, work backward, indirect reasoning (process of elimination), direct reasoning, write an equation.
3. The sum of the first n positive integers is $\dfrac{n\,(n+1)}{2}$.
4. We use 0 placeholders in our number system if we need to skip a value, such as 1024 skipping the hundreds value.
5. The base 5 number system has characters 0, 1, 2, 3, and 4, and each place value corresponds to a power of 5.
6. To convert a number from base 5 to base 10, we write it in expanded form. Example: $123_{\text{five}} = 1 \cdot 5^2 + 2 \cdot 5^1 + 3 = 38_{\text{ten}}$.
7. To convert a number from base 10 to base 5, we find the largest power of 5 smaller than the number and take out as many groups of that number as possible. We then repeat the process for each descending power of 5, and we do not skip one if it doesn't have any groups to remove. We have a visual way to draw this in the class examples.
8. The symbols for sets are $\in$ (is an element of), $\notin$ (is not an element of), $\subseteq$ (is a subset of), $\not\subseteq$ (is not a subset of), $\subset$ (is a proper subset of),
9. Set builder notation is a way to write sets in this manner: {form of elements | conditions}. Often, it is just $\{x \mid x \text{ satisfies some conditions}\}$.
10. The special sets are (1) The Natural Numbers: $\mathbb{N} = \{1, 2, 3, 4, ...\}$, (2) The Integers: $\mathbb{Z} = \{..., -4, -3, -2, -1, 0, 1, 2, 3, 4, ...\}$, (3) The Real Numbers: $\mathbb{R} = \{x \mid x \text{ is any number that can be written as a decimal}\}$, and there are others to discuss later.